NUCLEAR AND HYPER-NUCLEAR POINTS IN THE THEORY OF ABSTRACT SETS*

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1. *Introduction.* In his note, *Le théorème de* Borel *dans la theorie des ensembles abstraits,*† Fréchet considers the problem: determine the most general class (L) for which the theorem of Borel holds true. This class is found to be a class (S), that is, a class (L) in which the derived set of every set is closed. At the end of the note he calls attention to the fact that the stronger theorem of Borel-Lebesgue may not hold in a given class (S) and proposes the question: what is the most general class (L) for which we may state the theorem of Borel-Lebesgue? That such a class (L) be a class (S) is necessary but not sufficient.

This attracted the attention of R. L. Moore,‡ who showed by the aid of the theory of transfinite ordinals that the most general class (L) which admits the theorem of Borel-Lebesgue is a class (S) with the further property "every compact set is perfectly compact". The property *perfectly compact,* so named by Fréchet,§ is defined as follows. A set E is perfectly compact if every monotone sequence of subsets of E determines an element which is common to all the sets of the sequence or to their derived sets. A sequence of sets is monotone if of any two sets of the sequence one contains the other.

Later Fréchet,|| developing the theory of classes (V)

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† BULLETIN DE LA SOCIÉTÉ DE FRANCE, vol. 45 (1917), pp. 1–8. Called Fréchet, I hereafter.

‡ *On the most general class* (L) *of* Fréchet *in which the Heine-Borel-Lebesgue theorem holds true*, PROCEEDINGS OF THE NATIONAL ACADEMY OF SCIENCES, vol. 5 (1919), pp. 196–210.

§ *Sur les ensembles abstraits,* ANNALES DE L'ECOLE NORMALE (3), vol. 38 (1921), p. 342. Called Fréchet, II hereafter.

|| *Sur la notion de voisinage dans les ensembles abstraits,* BULLETIN DES SCIENCES MATHÉMATIQUES, (2), vol. 42 (1918), pp. 138–156. Called Fréchet, III hereafter.