The relation (9) follows by comparing (6) with the identity

$$\vartheta_0\vartheta_3^2 = \left[1 + 2\sum_{a=1}^{\infty} q^{a^2}(-1)^a\right]\left[1 + 4\sum_{n=1}^{\infty} q^n \xi(n)\right],$$

the second factor on the right being the algebraic expression of the well known theorem which gives the number of representations of n as a sum of two integer squares.

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NOTE ON A SPECIAL CONGRUENCE*

BY MALCOLM FOSTER

1. *Introduction.* Let S be any surface referred to its lines of curvature. With every point M of S we associate the trihedral of the surface, taking the x-axis of the trihedral tangent to the curve $v =$ const. We consider the congruence of lines l parallel to the z-axis, the normal to S, which pierce the xy-plane at the point $(\xi, \eta_1, 0)$.† The equations of l are $x = \xi$, $y = \eta_1$, and the coordinates of any point on l are

$$x = \xi, \quad y = \eta_1, \quad z = t, \tag{1}$$

where t is the distance on l measured from the point $(\xi, \eta_1, 0)$.

2. *Condition for a Normal Congruence.* If there be a surface normal to the congruence we must have

$$\delta z = dz + p_1 y dv - q x du = 0, \ddagger \tag{2}$$

for all values of $\frac{dv}{du}$. Using (1) equation (2) becomes

$$\frac{\partial t}{\partial u} du + \frac{\partial t}{\partial v} dv + p_1 \eta_1 dv - q\xi du \equiv 0; \tag{3}$$

hence

$$\frac{\partial t}{\partial u} - q\xi = 0, \qquad \frac{\partial t}{\partial v} + p_1\eta_1 = 0. \tag{4}$$

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† The notation used in this paper is the same as in Eisenhart's *Differential Geometry of Curves and Surfaces*; see pp. 166–176.

‡ Eisenhart, p. 170.