relations are

$$(3) \qquad h_1^{d_1} = 1, \quad h_2^{d_2} = 1, \quad \ldots, \quad h_r^{d_r} = 1$$

where those of the exponents $d_1, d_2, \ldots, d_r$ which are not 1 are the invariant factors of the matrix $|\gamma_{it}|$.

An operation of $\tilde{G}$ is of finite period if and only if it is in the group H generated by the operations $h_1, h_2, \ldots, h_r$ and the relations (3). Hence H contains all operations of $\tilde{G}$ of finite period. Moreover H is a finite group because it is commutative and generated by a finite number of operations each of finite period. The invariant factors of $|\gamma_{it}|$ are invariants of H (Kronecker, BERLINER MONATSBERICHTE, 1870, p. 885).

These invariant factors are what Tietze calls the Poincaré numbers of the group G. They are invariants of G because G determines the commutative group $\tilde{G}$ uniquely and $\tilde{G}$ determines the finite group H uniquely and H determines the invariant factors uniquely.

PRINCETON UNIVERSITY

ANALYTIC FUNCTIONS AND PERIODICITY*

BY J. F. RITT

This paper presents two theorems which show that the condition that a function be periodic can be analyzed, in different ways, into a set of requirements, from the satisfaction of only one of which, if the function is analytic, the periodicity of the function can be inferred. The theorems are

THEOREM A. *If $f(z)$ is a uniform analytic function, and if an $a > 0$ exists such that every z_1 at which $f(z)$ is analytic is the center of a circle of radius a on which a z_2 lies at which $f(z)$ is analytic and assumes the same value as at z_1, then $f(z)$ is periodic, and has a period of modulus a.*

* Presented to the Society May 3, 1924.