# INVARIANCE OF THE POINCARÉ NUMBERS OF A DISCRETE GROUP

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Some time ago Professor J. Nielsen of Copenhagen kindly pointed out to me that the last statement in § 30, page 142, of my *Colloquium Lectures on Analysis Situs* is incorrect; i. e., it is not true that "(12) is a necessary condition that (10) shall be a transformation to a new set of generators of $G$." For in solving equation (11) account must be taken of the generating relations.

This error invalidates the argument given in the next two sections for the invariance of the Poincaré numbers of a group $G$. It therefore seems worth while to point out the following rather obvious way of proving this invariance.

We are considering a group $G$ determined by a finite number of generators

$$g_1, g_2, \cdots, g_n$$

and a finite number of generating relations

$$g_1^{a_{i1}} g_2^{a_{i2}} \cdots g_n^{a_{in}} g_1^{b_{i1}} g_2^{b_{i2}} \cdots g_n^{b_{in}} \cdots g_1^{j_{i1}} g_2^{j_{i2}} \cdots g_n^{j_{in}} = 1 \qquad (i = 1, 2, \ldots, k). \tag{1}$$

This group determines uniquely a commutative group $\tilde{G}$ defined by the condition that it has $n$ generators subject to (1) and the condition that all operations of the group be commutative. In view of the commutativity, the generating relations of $\tilde{G}$ reduce to

$$g^{\gamma_{i1}} g^{\gamma_{i2}} \cdots g^{\gamma_{in}} = 1 \qquad (i = 1, 2, \ldots, k) \tag{2}$$

where

$$\gamma_{it} = a_{it} + b_{it} + \cdots + j_{it}.$$

For the group $\tilde{G}$, a new set of generators $h_1, h_2, \ldots, h_n$ can be found, as explained in § 31, for which the generating