THE SCIENTIFIC WORK OF JOSEPH LIPKA

BY W. C. GRAUSTEIN

Joseph Lipka, Associate Professor of Mathematics at the Massachusetts Institute of Technology, died January 15, 1924, in his forty-first year and the sixteenth year of his service at the Institute. He was a mathematician of proven worth, intensely interested in his science, and unremitting in his efforts to forward its progress. The last few years of his life were especially remarkable for their productivity. During this time he contributed what would normally be considered, both in volume and value, as the creditable work of a decade, and gave abundant promise of continued fruitfulness. That he has been taken in his prime is indeed a source of great regret and a distinct loss to mathematics.

During Lipka's student days at Columbia, Professor Kasner was busily applying differential geometry to dynamics and developing, in euclidean spaces of two and three dimensions, the geometric properties of dynamical trajectories and related systems of curves. It was this field of investigation in which Lipka was later initiated and did his work for the doctorate. His thesis consisted in a generalization, to a curved space of n dimensions, of results obtained by Kasner concerning a natural family of curves, that is, a family of trajectories in a conservative field of force corresponding to a given constant of energy, or more generally, any family of extremals resulting from a problem in the calculus of variations of the form,

$$\int e^{\varphi} ds = \text{minimum}, \tag{1}$$

where ds is the linear element of the space in question, and φ is a function of the coordinates of the space.

Lipka's later work, though largely confined to the one field, falls naturally into two periods. The earlier of these sees the generalization of many of Kasner's theories, first to ordinary surfaces and then to curved spaces of n dimensions. Noteworthy is the exhaustive study of dynamical trajectories on a surface for any positional field of force, and also the investigation of the geometrical properties of trajectory and related systems in Riemannian n-space. But perhaps the most important achievement of this period was the paper which established the validity, in all cases, of the Thomson-Tait criterion for a natural family. According to the theorem of Thomson and Tait, the ∞^{n-1} curves of a natural family meeting an arbitrary hypersurface orthogonally form a normal hypercongruence, i. e., admit of ∞^1 normal hypersurfaces. That this property is characteristic of natural familes (for $n > 2$) had been proved by Kasner for a euclidean space of three dimensions. In the paper now