THE JACOBIAN OF A CONTACT TRANSFORMATION*

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The equations

$$(1)\qquad x_1 = X(x, z, p),\quad z_1 = Z(x, z, p),\quad p_1 = P(x, z, p),$$

where X, Z, and P are functions of class C'', represent a transformation of line-elements in the xz plane to line-elements in the x_1z_1 plane. With Lie we shall define every transformation in x, z, p, which leaves the Pfaff differential equation

$$(2)\qquad dz - pdx = 0$$

invariant, as a contact transformation of the xz plane to the x_1z_1 plane. Hence the equations (1) must satisfy an identity of the form

$$(3)\qquad dz_1 - p_1dx_1 = \varrho(dz - pdx),$$

where ϱ is a function of x, z, and p alone.

The following relations connecting X, Z, P, and their partial derivatives are easily obtained:†

$$(4)\qquad \begin{cases} Z_x - PX_x = -p\varrho, \\ Z_z - PX_z = \varrho, \\ Z_p - PX_p = 0; \end{cases}$$

$$(5)\qquad \begin{cases} [XZ] = X_p(Z_x + pZ_z) - Z_p(X_x + pX_z) = 0, \\ [PX] = \varrho, \text{ and } [PZ] = \varrho P. \end{cases}$$

The jacobian of transformation (1) is

$$(6)\qquad J = \begin{vmatrix} X_x & X_z & X_p \\ Z_x & Z_z & Z_p \\ P_x & P_z & P_p \end{vmatrix}.$$

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* Presented to the Society, December 1, 1923.

† Lie und Scheffers, *Geometrie der Berührungstransformationen*, p. 68, Chap. 3.