BIANCHI'S DIFFERENTIAL GEOMETRY

Lezioni di Geometria Differenziale. Terza edizione interamente refatta in due volumi. Vol. I. By L. Bianchi. Pisa, Spoerri, 1921. IV + 806 pp.*

The first and second editions of this classic, whether in Italian or German, are so well known to all students of differential geometry, or should be, that an extended review of this edition would be an anachronism. When one turns over the pages he is reminded of the earlier editions, but when he compares it with the latter he observes much that is new in the development of the topics formerly treated. And so he decides that hereafter, when he wants to see what Bianchi says about a certain subject (and a student of differential geometry cannot afford not to do so), he will use the new edition.

The first ten chapters of the second editions, both Italian and German, are retained in this volume, but they have been developed to such an extent that they occupy half again as many pages.

The problem of reducing the linear element of a surface to the form $ds^2 = du^2 + 2\cos\omega\, du\, dv + dv^2$ is called the *problem of Tschebychef* by Bianchi, and the parametric curves the *lines of Tschebychef*. This problem was discussed briefly on page 401 of Volume 2 of the old edition, but the new treatment is more extensive. In fact, it is shown that such lines exist on any surface, the degree of arbitrariness being that of two arbitrary functions of one variable each (p. 159). More recently Bianchi† has proved that the tangents to either family of these lines, say $v = \text{const.}$, at points of any curve of the other family are parallel in the sense of Levi-Civita with respect to the latter curve; moreover, this is a characteristic property of the lines of Tschebychef.

In Chapter V it is shown that the Christoffel symbol of the second kind formed with respect to the second fundamental form is the arithmetic mean of the corresponding symbols formed with respect to the linear element of the surface and of its spherical representation. Also there are certain results concerning spaces of constant Riemannian curvature of n dimensions (p. 259).

In 1869 Dini solved the problem of geodesic representation of two surfaces upon one another. In 1896 Levi-Civita extended the problem to spaces of any order. The results of Dini and their application to

* The first German edition was reviewed by J. K. Whittemore in this BULLETIN, vol. 7 (1901), pp. 431–442, and the second German edition by L. P. Eisenhart in vol. 18 (1912), pp. 411–418.

† BOLLETTINO UNIONE MATEMATICA ITALIANA, vol. I (1922), pp. 11–16.