ON THE APPLICATION OF THE THEORY OF IDEALS TO DIOPHANTINE ANALYSIS*

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1. *Introduction.* About three years ago† Professor Dickson stated a certain conjectured theorem, and he has recently published a proof of it.‡

After having examined a proof of the same theorem by the author of this article, Professor Dickson suggested the investigation of a more general equation than the one which he had considered, and the following pages contain the results of this investigation.

2. *Rings.* Let us consider any algebraic number field $k(\theta)$ of degree n. Let $\gamma_1, \gamma_2, \ldots, \gamma_n$ be a fundamental system of integers of $k(\theta)$ so that every integer of the field can be represented by the fundamental form

$$x_1\gamma_1 + x_2\gamma_2 + \cdots + x_n\gamma_n, \tag{1}$$

in which the $x_1, x_2, \ldots, x_n$ are rational integers.

By a *ring* R in $k(\theta)$ we understand a set of integers which is closed with respect to addition, subtraction, and multiplication, and which contains the rational integers. Let $\varrho_1, \varrho_2, \ldots, \varrho_n$ be a fundamental system of R. As above, we shall call

$$x_1\varrho_1 + x_2\varrho_2 + \cdots + x_n\varrho_n \tag{2}$$

the *fundamental form* of R. Every element of R is represented once and only once by (2) when the $x_1, x_2, \cdots, x_n$ are given rational integral values.

Since $\varrho_1, \varrho_2, \ldots, \varrho_n$ are integers in $k(\theta)$, they can be represented by (1), and we shall suppose that

$$\varrho_i = r_{i1}\gamma_1 + r_{i2}\gamma_2 + \cdots + r_{in}\gamma_n, \qquad (i = 1, 2, \ldots, n). \tag{3}$$

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* Presented to the Society, December 29, 1923.

† L. E. Dickson, *A new method in Diophantine analysis*, this BULLETIN, vol. 27, No. 8 (May, 1921), p. 353.

‡ L. E. Dickson, *Integral solutions of $x^2 - my^2 = zw$*, this BULLETIN, vol. 29, No. 10 (Dec., 1923), p. 464.