and conditions (3*a*), (3*b*) become

$$(7) \qquad A + D < B + C, \qquad AD = 0.$$

Hence we find that *the most general existent boolean relation R which satisfies* (1) *is given by* (2) *and* (7).

Our main results may also be stated in the following form: *The totality of transitive universal relations in a boolean algebra is given by* (4). *The totality of existent transitive universal relations is given by*

$$Axy + Bxy' + Cx'y + Dx'y' = 0, \quad A + D < B + C, \quad AD = 0.$$

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ON CERTAIN QUINARY QUADRATIC FORMS*

BY E. T. BELL

1. *Introduction.* Except the classical theorems on the total number $N_5(n)$ of representations of the integer n as a sum of five integer squares, no explicit results on numbers of representations in quinary quadratic forms seem to have been obtained.† In general $N_5(n)$ is not expressible as a function of the real divisors alone of a single integer, but when n is a square, $N_5(n)$ is so expressible. This remarkable fact was found inductively by Stieltjes for $N_5(p^2)$, p prime, and proved for $N_5(n^2)$ by Hurwitz,‡ who showed that if $n = 2^\alpha m$, m odd, $N_5(n^2) = 10\zeta_3(2^\alpha)H(m)$, where

$$H(m) = [\zeta_3(p^a) - p\zeta_3(p^{a-1})][\zeta_3(q^b) - q\zeta_3(q^{b-1})]\dots,$$

$\zeta_r(n)$ being the sum of the rth powers of all the divisors of n, and $m = p^a q^b \dots$ the prime factor resolution of m; by convention $H(1) = 1$. In the course of his proof he showed that

$$\zeta_1(m^2) + 2\zeta_1(m^2 - 2^2) + 2\zeta_1(m^2 - 4^2) + \cdots = H(m).$$

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* Presented to the Society, December 27, 1923.

† Cf. Bachmann, *Zahlentheorie*, vol. 4, pp. 565-594.

‡ COMPTES RENDUS, vol. 98 (1884), pp. 504-7; cf. Dickson's *History of the Theory of Numbers*, vol. 2, p. 311. For quadratic forms in $n > 4$ variables, cf. ibid., vol. 3, chap. XI.