PROBLEMS IN INVOLUTORIAL TRANSFORMATIONS OF SPACE*

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1. *Introduction.* A report of great value and of general interest was presented to this Society at the Chicago meeting of April, 1922. While only a limited number had the opportunity to hear Professor Coble on that occasion, fortunately his message has reached a much wider public, by appearing in this BULLETIN (vol. 28 (1922), pp. 329–364). On account of the wider purpose there in hand, it was impossible to treat in detail all the many ramifications of the theory, or to show all their interrelations. My present purpose is to comment more fully on one narrow phase of this report, namely, that of involutions.

In the plane the problem is almost completely solved. It was shown by Bertini(1)† that there are four types to one of which every plane involutorial transformation can be reduced; they are the harmonic homology (H), the Geiser(2) (G) of order 8 with 7 triple points, the Bertini(1) (B) of order 17 with 8 six-fold points, and the Jonquières(3) (J) of order n with one fundamental point of order $n-1$ and $2n-2$ simple ones. Of these, all but the last are individual types, but (J) exists for every positive integral value of n. For special (J), JH is also an involution, but this can always be reduced to an H. Moreover, all these involutions are rational, that is, the pairs of conjugate points can be mapped rationally upon a plane (x') such that between (x') and the given plane (x) there exists an algebraic (1, 2) point correspondence.

Thus, that associated with (H) may be expressed in the form $x_2'x_3-x_3'x_2=0$, $x_1'x_2x_3-x_2'x_1^2=0$. The invariant

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†The numbers refer to the papers listed at the end of this article.