that $a\mu_{r^{mg}} \geqq 1$, where r^{mg} is the representative element of any class $\mathfrak{P}^{mg}$ to which p belongs. Using such a function μ in the condition (1b) on the developmental system and noting that the sequence $\{\mu_{0p_{2n_k}}\}$ is bounded, we see that for every e there is an m_e such that for $m \geqq m_e$ and for every k the value $\theta_{p_{2n_k}}{}^m$ does not exceed e. But this affords a contradiction to the conclusion reached above that

$$L_m(L_k\theta_{p_{2n_k}}{}^m) = 1.$$

Therefore the hypothesis that p is not a limit of the sequence $\{p_{2n}\}$ is contrary to fact.

These considerations may be extended to the infinite developments of Chittenden.*

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ON THE ADJOINT OF A CERTAIN MIXED EQUATION.

BY DR. R. F. BORDEN.

(Read before the American Mathematical Society December 30, 1919.)

CONSIDER a function of the form

(1) $F\{f(x)\} = \Delta f'(x) + a(x)f'(x) + b(x)\Delta f(x) + c(x)f(x),$

where $a(x)$, $b(x)$, and $c(x)$ are analytic functions of x, also $\Delta f(x) = f(x+1) - f(x)$, and $f'(x) = (d/dx)f(x)$. We will say that $G\{g(x)\}$ is the adjoint of $F\{f(x)\}$ if $G\{g(x)\} = 0$ is the condition that

$$\int \Sigma\, g(x)F\{f(x)\}dx = \Sigma M(x) + \int N(x)dx, \tag{2}$$

where Σ denotes an inverse of Δ.

This condition (2) is satisfied if

$$g(x)F\{f(x)\} = \frac{d}{dx}M(x)dx + \Delta N(x), \tag{3}$$

* Cf. "Infinite developments and the composition property $(K_{12}B_1)_*$ in general analysis," by E. W. Chittenden, *Rendiconti del Circolo Matem. di Palermo*, vol. 39 (1915), p. 21, §§ 19 and 21.