PONCELET POLYGONS IN HIGHER SPACE.

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(Read before the American Mathematical Society December 30, 1919.)

LET there be given a linear projective space of $2n$ dimensions. A point of the space may be denoted by P and its dual figure by P'. Thus a P' is a linear space of $2n - 1$ dimensions.

The totality of P's in the space is infinity to the order $2n$, and the totality of P''s is of course of this same order. We shall select from these totalities a Q_n and a Q_n' respectively, general quadratic loci of infinity to the order n of elements, where Q_n consists of P's, and Q_n' of P''s.

For Q_n and Q_n' not in specialized relation to each other we have a two-two correspondence of the following form: Each P of Q_n meets two P''s of Q_n', and each P' of Q_n' meets two P's of Q_n. Starting with any point of Q_n, a succession of points of Q_n is determined, where furthermore consecutive points of the sequence may be joined by lines. The succession of lines forms then a single "broken line" as this term is used in projective geometry. It may or may not happen that the broken line closes into a polygon. Except for degenerate cases corresponding to coincident P's or P''s, and it being supposed that Q_n and Q_n' are not degenerate, it may be proved that the closure of the broken line is determined by the relative positions of Q_n and Q_n' and is independent of the element selected as initial.

This may be called a theorem of Poncelet polygons in higher spaces. For $n = 1$, the theorem is the usual one.

It should be emphasized that the case for $n > 1$ is not the logical equivalent of the case for $n = 1$, since there are n independent parameters in any case. The proof of the theorem is immediate by reference to general theorems on algebraic correspondences or to theta functions, the quadratics Q_n and Q_n' determining theta functions of genus n, and affording one of the simplest illustrations of their character.

A second generalization and one which applies to three-space is to spaces of $2n - 1$ dimensions generally, $n > 1$, the P, P', Q_n, Q_n' being as above. Any P' of Q_n' may be viewed