Taking each r equal to unity, we have the interesting special case

$$X \leqq \max\{1 + |\beta_1|, 1 + |\beta_2|, \cdots, 1 + |\beta_{n-1}|, |\beta_n|\}.$$

Again, it is easy to show also that

$$X \leqq \max\{|\alpha| + |\beta|, |\beta| + |\gamma|, \cdots, |\lambda| + |\mu|, |\mu|\},$$

where the quantities $\alpha, \beta, \cdots, \mu$ are defined by the relations

$$\alpha = \beta_1, \beta^2 = \beta_2, \beta\gamma^2 = \beta_3, \beta\gamma\delta^2 = \beta_4, \cdots, \beta\gamma\cdots\lambda\mu^2 = \beta_n.$$

Through other special choices of the quantities r numerous rather elegant special inequalities may be obtained, several of which are given explicitly by Kojima (loc. cit.).

In case some of the coefficients of equation (1) are zero it may be preferable to employ the second italicized theorem in § 6 and apply to it the principal theorem of this section (or the special cases of the latter).

It is clear that other general formulas may be found for upper bounds to X by employing a sequence of equations each of which is linear or quadratic, such sequence arising by the repeated application of the theorem of this section.

UNIVERSITY OF ILLINOIS,
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THE SOLUTION OF THE WAVE EQUATION BY MEANS OF DEFINITE INTEGRALS.

BY PROFESSOR H. BATEMAN.

THE wave equation

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 V}{\partial t^2} \tag{1}$$

is the oldest member of the family of partial differential equations, and although he was born without the second and third terms he soon acquired these and played a prominent part in mathematical physics at a time when very few partial differential equations had become famous. With the advent of the mathematical theory of elasticity and Maxwell's electromagnetic theory of light he gained a new lease of life and more