

special theorems accompanying it, is developed in Parts II and III. In Part IV curvilinear coordinates are used in the problems and theorems of space curves and surfaces. Here is also included the application of line coordinates to the general theory of surfaces of the second order.

A list of 136 exercises, to some of which is added the suggestion that they be solved geometrically, is given at the close of the book.

It is to be regretted that the volume does not possess the high typographical tone and finish which characterize Scheffer's text.

ERNEST W. PONZER.

Untersuchungen über die im Schlusswort des Lie'schen Werkes "Geometrie der Berührungstransformationen" angedeuteten Probleme. Von PHILIPP ENGELHARDT (Inaugural-Dissertation, Würzburg). Leipzig, B. G. Teubner, 1910. 65 pp.

THE closing lines of Lie's *Geometrie der Berührungstransformationen* forecasted that a second volume, which never appeared, would treat the theory of differential equations from the standpoint of contact transformations and continuous groups. One may well hope that this field will soon receive its fair share of attention and be extensively developed beyond the elements established by Lie.

The thesis under review might be classed with fundamental literature having this tendency. The author's methods are, for the most part, those presented by Lie in chapter 14. The integral surfaces of a non-linear partial differential equation of the first order, $F(x, y, z, p, q) = 0$, bear an aggregate of ∞^3 exceptional curves called characteristics (*Geometrie der Berührungstransformationen*, pages 261, 498). Lie's summary of his last chapter is in the form of statements of six problems which are there solved. These are determinations of differential equations $F = 0$ defined by definite conditions imposed upon the characteristics or upon the normals to integral surfaces. Problem 2 is the determination of the equations $F = 0$ whose characteristics are lines of curvature upon all integral surfaces. Problem 4 is the determination of the equations $F = 0$ whose integral surfaces have for normals precisely lines of a given line complex. This thesis is devoted to twelve problems obtained by combining Lie's six problems. Thus problem (2, 4) of the thesis is the deter-