We shall note a few additional properties of the function we have obtained.

In addition to being single valued, $F(x)$ assumes a given value but once. We can thus regard it as giving a one-to-one transformation of the interval (0, 1) into itself, which is everywhere discontinuous. At every point save $x = \frac{1}{2}$ the function has no limit; that is, every point, except $x = \frac{1}{2}$, is a point of discontinuity of the second kind. It is also apparent that both the greatest and least values approached at a point are continuous functions.

INDIANA UNIVERSITY,
May, 1914.

PROOF OF THE CONVERGENCE OF POISSON'S INTEGRAL FOR NON-ABSOLUTELY INTEGRABLE FUNCTIONS.

BY DR. W. W. KÜSTERMANN.

IN the following pages I propose to give a proof of the

THEOREM: *If $f(x)$ is a real, periodic function, of period 2π, which in the interval $(0, 2\pi)$ has a proper or improper integral in the sense of Lebesgue, Harnack-Riemann, or Harnack-Lebesgue-Hobson,* then*

$$\lim_{r \to 1} \frac{1}{2\pi} \int_{-\pi}^{\pi} f(\alpha) \frac{1 - r^2}{1 + r^2 - 2r \cos (\alpha - x)} d\alpha = \lim_{t \to 0} \tfrac{1}{2}[f(x + t) + f(x - t)]$$

at every point x where the limit on the right hand side exists.

This theorem includes in particular the case where $f(x)$ remains finite—disposed of by Schwarz,† and the case where $f(x)$ becomes infinite at an infinite number of points, but has an *absolutely* convergent improper integral—discussed by Hobson and others.‡ Moreover, it goes farther, in that it

* For these definitions see Hobson, Theory of Functions of a Real Variable, Cambridge, 1907.

† Schwarz, Math. Abhd., vol. 2, pp. 144 and 175.

‡ Most completely by Hobson, Theory of Functions of a Real Variable, p. 719; cf. also Bôcher, *Ann. of Math.*, 2d ser., vol. 7, p. 81; Fatou, *Acta Math.*, vol. 30, p. 335; Picard, Traité d'Analyse, 2d ed., vol. 1, p. 268; Forsyth, Theory of Functions, 2d ed., p. 450.