ON A GENERALIZATION OF A THEOREM OF DINI ON SEQUENCES OF CONTINUOUS FUNCTIONS.

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WE propose in this note to give a generalization of the following theorem of Dini*: "If a monotonic sequence of functions continuous on a closed interval converges to a continuous function, the convergence is uniform."

The double sequence analogue of this theorem proves to be of importance in our generalization. We embody it in the following

LEMMA. *If a double sequence a_{mn} is monotonic non-decreasing in m for every n, and if $L_mL_na_{mn} = L_nL_ma_{mn}$, all the limits being supposed to exist, then L_ma_{mn} and L_na_{mn} converge uniformly and the double limit $L_{mn}a_{mn}$ exists and is equal to the iterated limits.*

The proof of the uniformity of convergence of L_ma_{mn} is part of Theorem I of the paper by the author in the *Annals of Mathematics*, series 2, volume 14, page 81. This uniformity has as a direct consequence the existence of the double limit equal to the iterated limits, which in turn implies the uniformity of L_na_{mn}.

For the purposes of generalization, consider a class $\mathfrak{D}$ of elements unconditioned excepting for the existence within the class of some definition of limit, i. e., some means of determining whether a sequence of elements has a limit and what this limit is.† Then it is possible to define the concepts *limiting element, closed and compact* relative to subclasses $\mathfrak{R}$ of $\mathfrak{D}$.‡ Also if μ is a real-valued function on $\mathfrak{R}$, we can define the notion of *continuity,* as well as *equal continuity,* as applied to a set of functions. In such a situation we are able to state the following

THEOREM. *If $\mathfrak{R}$ is a closed and compact subclass of $\mathfrak{D}$, and μ_{nr} is a monotonic sequence of functions continuous on $\mathfrak{R}$ and*

* Cf. Dini-Lüroth-Schepp: Theorie der Funktionen, p. 148.

† Cf. *Amer. Journ. of Mathematics,* vol. 34, p. 241.

‡ Cf. Fréchet: "Sur quelques points du calcul fonctionnel," *Rendiconti del Circolo Matematico di Palermo,* vol. 22 (1906), pp. 6, 7, 11.