## 1914.] INFINITE REGIONS OF VARIOUS GEOMETRIES.

11. In the American Journal of Mathematics, volume 34 (1912), page 173, Mr. Schweitzer has shown how to generate a quasi-four-dimensional geometry  ${}^4R_{4}^{(0)}$  by adjoining a point tactically to the system  ${}^{3}R_{3}$  and assuming the axiom " $\alpha R\beta \gamma \delta \epsilon$  implies  $\delta R\epsilon\alpha\beta\gamma$ " which ensures that the generating relation is alter-The resulting system is sufficient for the usual threenating. dimensional projective geometry if an axiom expressing Dedekind continuity (suitably modified for projective geometry) is added. This geometry  ${}^{4}R_{4}^{(0)}$  may be regarded as underlying a system of four-dimensional simplexes inscribed in a hypersphere. In the Archiv der Mathematik und Physik, volume 21 (1913), page 204, E. Study has remarked that the figure of five ordered real points, no four of which are coplanar, has a (single) property, "signatur" (+) or (-), which is not disturbed by positive real collineations. It seems simpler and altogether more convenient to regard Study's figure of five points with positive or negative "signatur" as a sensed simplex in quasifour space as indicated above.

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## THE INFINITE REGIONS OF VARIOUS GEOMETRIES.

## BY PROFESSOR MAXIME BÔCHER.

## (Read before the American Mathematical Society, September 8, 1913.)

Most geometers are now conscious that the introduction of points at infinity in such a way that in plane geometry they form a line, in three-dimensional geometry a plane, is, to a large extent, an arbitrary convention; but few of them would probably admit that this remark has much practical importance (except in so far as they might regard any question concerning the logical foundation of geometry as having practical importance) since the convention here referred to is commonly regarded as being the only desirable one. It is the object of the present paper to point out more explicitly and in greater detail than has, to my knowledge.