

6. The curve of S_4 of which the double curve of $F'_{\mu\nu}$ is the projection is a complete intersection, but such is not the case for the double curve itself. In this respect the case where $\mu = \nu = 3$ is instructive. For then it is found that $t = 6\binom{3}{3} = 6$, $p_g = p_n = 5$,

$$\pi = 5 \cdot 18 + 5 - \binom{8}{3} - 2 \cdot 6 + 1 = 28,$$

$$h = \frac{1}{2}3 \cdot 3 \cdot 2 \cdot 2 = 18 = 6 \cdot 3 = 9 \cdot 2.$$

The complete intersection of an F_9 and an F_2 with six triple points is of genus 63, while the intersection of an F_6 and an F_3 with six triple points is of genus 28, and yet the curve with the above characteristics cannot lie on an F_3 , for then it would meet F'_9 in a curve of order 36. If $F_{\nu(\mu-1)(\nu-1)}$ is the intersection of V_ν and $V_{(\mu-1)(\nu-1)}$, then the curve in S_4 is on $F_{\nu(\mu-1)(\nu-1)}$, the residual intersection being of order $\mu\nu(\mu-1)(\nu-1)^2$, and its genus p_1 is found by remarking that if we project from another center this curve will be residual of one of order $\mu\nu(\mu-1)(\nu-1)$ and genus π , so that

$$\pi_1' = \pi + \frac{1}{2}\mu\nu(\mu-1)(\nu-1)(\nu-2)[\mu\nu + \nu(\mu-1)(\nu-1) - 4].$$

Thus, above, the residual curve is of order 72 and genus 397.

LINCOLN, NEB.,
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GEOMETRICAL OPTICS.

The Principles and Methods of Geometrical Optics, especially as Applied to the Theory of Optical Instruments. By JAMES P. C. SOUTHALL. New York, The Macmillan Company, 1910. xxiii+626 pp. with 170 figures.

THAT mathematicians and physicists have left the field of geometrical optics so largely to the scientific experts of the best firms of optical engineers may be but one of the signs of our ever increasing specialization and its accompanying narrowing of interests. Yet the association of such names as Euler, Fermat, Gauss, Hamilton, Kummer, Moebius, Sturm shows that once mathematicians contributed largely to the subject and were inspired by it; a similar state of affairs is true in regard to physicists.

From an impartial viewpoint it can hardly be gainsaid that there are at present more points of contact between geometrical