DEFINITE INTEGRALS CONTAINING A PARAMETER.

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A FUNCTION $f(\alpha, x)$ is defined for each pair of values of α and x in the closed region $0 \leqq \alpha \leqq 1$ and $0 \leqq x \leqq 1$. For each value of α in the interval $(0, 1)$ the function $f(\alpha, x)$ is an integrable function of x according to Riemann's definition. A function $F(\alpha)$ is thus defined by the equation

$$F(\alpha) = \int_0^1 f(\alpha, x)dx.$$

The problem considered in this paper is one of uniform convergence; namely, the determination of the conditions to be imposed on the function $f(\alpha, x)$ in order that corresponding to any positive number ϵ there exist a number δ independent of α such that

$$\text{(I)} \qquad \left| F(\alpha) - \sum_{i=0}^{i=n} f(\alpha, \xi_i)(x_i - x_{i-1}) \right| < \epsilon,$$

$$(x_0 = 0, \quad x_n = 1, \quad x_{i-1} \leqq \xi_i \leqq x_i)$$

for $(x_i - x_{i-1}) < \delta$.

Closely associated with this problem of uniform convergence are, at any rate, two others which lend interest to it. Of these, one is the problem concerning the continuity of $F(\alpha)$. Under the assumption that $f(\alpha, x)$ is a continuous function of α for each value of x, a necessary and sufficient condition that $F(\alpha)$ be a continuous function of α follows from the theory developed. The conditions under which the roots of the equation $F(\alpha) = 0$ are limiting points of the roots of the sequence of equations

$$\sum_{i=0}^{i=n} f(\alpha, \xi_i)(x_i - x_{i-1}) = 0$$

as n becomes infinite is the second problem.

The absence of continuity conditions does not preclude the existence of the inequality (I).