SHORTER NOTICES.

Arithmétique Graphique. Introduction a l'Etude des Fonctions Arithmétique. Par G. ARNOUX. Paris, Gauthier-Villars, 1906. xx + 225 pp.

THIS is the second of two volumes published by the author under the general title Arithmétique graphique. The first of these appeared in 1894 with the special title Les espaces arithmétiques hypermagiques. It has to do with magic squares, cubes, etc., but its methods are those of the theory of numbers. While it was still in press an abstract of it was presented to the Société mathématique de France by C. A. Laisant,* who was so impressed with its methods that he made the following prophecy: "Il y a en effet une telle originalité, une telle puissance d'invention dans les méthodes dont il s'agit, que je serais bien étonné si l'arithmétique n'arrivait pas quelque jour à les utiliser, soit pour obtenir des démonstrations plus simples de vérités connues déjà, soit pour découvrir des vérités nouvelles."

The starting point of Arnoux's method is the definition of a modular arithmetic space. A one-dimensional arithmetic space of modulus m consists of the m points of a straight line whose coordinates are $0, 1, 2, \cdots, m-1$. A two-dimensional arithmetic space consists of the m^2 points of a plane whose cartesian coordinates are the m^2 pairs of the integers $0, 1, 2, \cdots, m-1$. The extension to three or more dimensions is obvious. An arithmetic space of modulus m may be thought of as a *finite geometry* that corresponds to the *finite algebra* of integers modulo m. An arithmetic straight line in a two-dimensional arithmetic space consists of the m points whose coordinates satisfy a linear congruence of the form $ax + by + c \equiv 0 \pmod{m}$.† The points of such a line do not in general all lie on a euclidean straight line but there is a close connection between the two. For those points of a euclidean plane whose coordinates are integers may be arranged in an infinite number of arithmetic spaces of modulus m by reducing the coordinates modulo m. A euclidean straight line determined by two points A and B of one of these modular arithmetic spaces will

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† Arnoux gives an equivalent definition in terms of the function $ax + by$ and the modulus m.