A TABLE OF MULTIPLY PERFECT NUMBERS.

BY PROFESSOR R. D. CARMICHAEL.

(Read before the American Mathematical Society, February 23, 1907.)

A MULTIPLY perfect number is one which is an exact divisor of the sum of all its divisors, the quotient being the multiplicity.* The object of this paper is to exhibit a method for determining all such numbers up to 1,000,000,000 and to give a complete table of them. I include an additional table giving such other numbers as are known to me to be multiply perfect.

Let the number N, of multiplicity m $(m > 1)$, be of the form

$$N = p_1^{a_1} p_2^{a_2} \cdots p_n^{a_n},$$

where $p_1, p_2, \cdots, p_n$ are different primes. Then by definition and by the formula for the sum of the factors of a number, we have

$$(1)\quad m = \frac{p_1^{a_1} + p_1^{a_1-1} + \cdots + p_1 + 1}{p_1^{a_1}} \cdots \frac{p_n^{a_n} + p_n^{a_n-1} + \cdots + p_n + 1}{p_n^{a_n}}.$$

Hence

$$(2)\qquad m < \frac{p_1}{p_1 - 1} \cdot \frac{p_2}{p_2 - 1} \cdots \frac{p_n}{p_n - 1}.$$

These formulas will be of frequent use throughout the paper.

Since $2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23 = 223{,}092{,}870$, multiply perfect numbers less than 1,000,000,000 contain not more than nine different prime factors; such numbers, lacking the factor 2, contain not more than eight different primes; and, lacking the factors 2 and 3, they contain not more than seven different primes.

First consider the case in which N does not contain either 2 or 3 as a factor. By equation (2) we have

$$(3)\qquad m < \tfrac{5}{4} \cdot \tfrac{7}{6} \cdot \tfrac{11}{10} \cdot \tfrac{13}{12} \cdot \tfrac{17}{16} \cdot \tfrac{19}{18} \cdot \tfrac{23}{22} = \tfrac{676039}{331776}.$$

Hence $m = 2$; moreover seven primes are necessary to this value. Now, $5 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23 = 37{,}182{,}145$. Therefore, since we are not to consider numbers greater than 1,000,000,000,

* The name "multiply perfect" was introduced by Lehmer, *Annals of Math.*, ser. 2, vol. 2, p. 103.