NOTE ON FOURIER'S CONSTANTS.

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(Read at the Preliminary Meeting of the Southwestern Section of the American Mathematical Society, December 1, 1906.)

A FUNCTION $f(x)$ real and single valued, bounded, and in the sense of Riemann integrable from 0 to 2π gives rise to the Fourier's series

$$(1)\quad \tfrac{1}{2}a_0 + \sum_{k=1}^{\infty}(a_k \cos kx + b_k \sin kx) = \tfrac{1}{2}\sum_{k=-\infty}^{+\infty}(a_k \cos kx + b_k \sin kx),$$

where

$$(2)\quad a_k = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos kx\,dx, \qquad b_k = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin kx\,dx. \qquad (k = 0, \pm 1, \cdots)$$

Hurwitz * calls the constants a_k, b_k the Fourier's constants of the integrable function $f(x)$, and for the realm of integrable functions he studies the theory of Fourier's constants instead of the theory of Fourier's series which are convergent only under conditions.

The realm of integrable functions is a realm of integrity, *i. e.*, the sum, the difference, and the product of two integrable functions is integrable. The constants a_k, b_k, or in more explicit notation a_{kf}, b_{kf}, depend in a linear distributive way on the function $f = f(x)$. Further, the constants a_{kh}, b_{kh}, for the product $h(x) = f(x)\,g(x)$ of two functions $f(x)$, $g(x)$ are determinable from the constants a_{kf}, b_{kf} and a_{kg}, b_{kg} of these functions. In fact, from (2) we have

$$(3)\quad a_{0f} = \frac{1}{\pi}\int_0^{2\pi} f(x)dx,$$

$$(4)\quad a_{kf} = a_{0,f(x)\cos kx}, \qquad b_{kf} = a_{0,f(x)\sin kx},$$

* Hurwitz, "Ueber die Fourierschen Konstanten integrierbarer Funktionen," *Math. Annalen*, vol. 57 (1903) pp. 425–446, and vol. 59 (1904), p. 553.