NOTE ON CONJUGATE POTENTIALS.

BY PROFESSOR O. D. KELLOGG.

(Read before the American Mathematical Society, September 3, 1906.)

IF $u(r, \vartheta)$ is a potential function of the unit circle and $v(r, \vartheta)$ its conjugate, and if $f(\vartheta)$ and $g(\vartheta)$ are the values approached by these functions as $r \doteq 1$, the following relations given by Hilbert * hold:

$$f(\vartheta) = \frac{1}{2\pi}\int_0^{2\pi} g(\phi) \cot \tfrac{1}{2}(\vartheta - \phi)d\phi + \frac{1}{2\pi}\int_0^{2\pi} f(\phi)d\phi,$$

$$g(\vartheta) = -\frac{1}{2\pi}\int_0^{2\pi} f(\phi) \cot \tfrac{1}{2}(\vartheta - \phi)d\phi + \frac{1}{2\pi}\int_0^{2\pi} g(\phi)d\phi,$$

where by the integration symbols in the first terms of the right hand sides the Cauchy principal value is meant. They give, to within an additive constant, the boundary values of a potential in terms of its conjugate.

They have been established for the case that $f(\vartheta)$ and $g(\vartheta)$ are integrable throughout and are continuous at all but a finite number of points and possess derivatives subject to the same conditions.† Because of the interest attaching to them from their connection with potential theory, and as examples of integral equations of the first kind, ‡ it seems worth while to point out that they hold if only $f(\vartheta)$ and $g(\vartheta)$ are integrable from 0 to 2π and are such that

$$\int \frac{f(\vartheta_0 + \delta) - f(\vartheta_0)}{\delta} d\delta \quad \text{and} \quad \int \frac{g(\vartheta_0 + \delta) - g(\vartheta_0)}{\delta} d\delta$$

are convergent when extended over an interval including $\delta = 0$ in its interior and this for all but a finite number of values of

* Vorlesungen über Potentialtheorie, Göttingen, winter semester, 1901–02.

† See *Math. Annalen*, vol. 58, p. 442; also my dissertation for the doctorate: "Zur Theorie der Integralgleichungen und des Dirichlet'schen Prinzips," Göttingen, 1902, p. 17.

‡ Hilbert: "Grundzüge einer allgemeinen Theorie der linearen Integralgleichungen," *Gött. Nachrichten*, 1904, p. 49. Encykl. d. Math. Wiss. II, A, 11, pp. 803, 816.