THE GROUP OF A TACTICAL CONFIGURATION.

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1. A tactical configuration, connected with the abelian group G_{p^m} of type $(1, 1, \cdots, 1)_m$ and serving to define the general linear homogeneous group H modulo p on m variables, has been given by Professor Moore.* As an obvious generalization, consider the configurations defining the various subgroups of H. The example that I give had its origin in the following problem:† Required the number N_n of all possible ways of separating the $2^{2n} - 1$ operators other than identity of $G_{2^{2n}}$ into $2^n + 1$ sets each of $2^n - 1$ operators, such that the operators of any set together with identity form a subgroup G_{2^n} and such that no two sets have a common operator. Here $G_{2^{2n}}$ is assumed to be an abelian group of type $(1, 1, \cdots, 1)_{2n}$; for example, the group of all linear transformations on $2n$ variables which multiply each variable by ± 1.

2. That such a separation is always possible is easily shown. The group of automorphisms of $G_{2^{2n}}$ may be taken concretely as the group of all linear homogeneous transformations modulo 2 on $2n$ variables. The latter contains ‡ a transformation S whose characteristic equation of degree $2n$ is irreducible modulo 2, so that S is of period $2^{2n} - 1$. In particular, an operator Σ of period $2^n - 1$ occurs. Let $I, a_1, b_1, a_1b_1, \cdots$ be the operators of a subgroup G_{2^n} of $G_{2^{2n}}$. A table of the operators of the latter may be formed with those of G_{2^n} in the first row and with the operators $I, a_2, b_2, \cdots$ of a second G'_{2^n} as multipliers. We may choose $\Sigma = \Sigma_1\Sigma_2$, where Σ_i permutes cyclically the $2^n - 1$ elements $a_i, b_i, a_ib_i, \cdots$, written in a suitable order. As the first set S_1 we take $a_1, b_1, a_1b_1, \cdots$; as the second set S_2 we take $a_2, b_2, \cdots$. To form the third set S_3, take any element, as a_1a_2, in neither S_1 nor S_2, and apply to it the powers of Σ; there result $a_1a_2, b_1b_2, \cdots$. To form the i^{th} set take any element not in $S_1, S_2, \cdots, S_{i-1}$ and apply to it the powers of Σ. In this way we obtain $2^n + 1$ sets with the desired properties.

* BULLETIN, vol. 2 (1895), pp. 33–43.

† For $n = 2$, see Burnside, Theory of Groups, p. 60, ex. 2; errata, p. xvi.

‡ Linear Groups, p. 236.