THE ANALYTIC THEORY OF DISPLACEMENTS.

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Introduction.

THE subject of displacements is connected with several branches of pure mathematics and can be approached from many sides. The most comprehensive point of view is, no doubt, to regard the subject as an illustration of the theory of groups, the group here involved being a particular case of the group of linear homogeneous transformations. Again, instead of the symbol of an infinitesimal transformation, we may use the matrix of the corresponding linear substitution, and by means of elementary properties of matrices much of the work may be shortened and simplified.

By a generalization of the ordinary notion, a displacement may be defined as a projective point transformation which does not alter distance as measured with reference to an absolute quadric locus, and which further is capable of being represented as the result of an infinite number of infinitesimal transformations of the same kind. The analytic representation of a displacement must be a linear transformation of the variables which leaves the absolute unaltered. Since by a suitable choice of real or imaginary coördinates the equation of the absolute may be expressed by means of the sum of the squares of the coördinates, the problem in its simplest algebraic form is reduced to finding the most general orthogonal transformation.

The main object held in view throughout this paper is to obtain the general parametric representation of a displacement in two or three dimensions on the basis of the definition given above, and the method adopted is to integrate the equations of infinitesimal transformation. The formulæ are developed in a natural manner from the beginning and in addition to elementary processes use is made of matrices and quaternions. Not much is said about the other kind of transformation, corresponding to an improper orthogonal matrix, that is, one whose determinant is negative, because the general transformation of this kind can be obtained by combining a suitable displacement with any particular reflexion, and accordingly the analytic