ON THE QUINTIC SCROLL HAVING THREE DOUBLE CONICS.

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IN his paper on quintic scrolls,* Professor Schwarz mentioned that such a scroll exists having three double conics (type IX of those of genus 0). It is now proposed to derive the equation of this surface and to study some of its properties.

A scroll having two double conics will in general be of order 8 and genus 1. In order to generate a quintic scroll the conics must therefore intersect in two points, one of which is a simple self-corresponding point of the (2, 2) correspondence between the points of the two conics, and the other must be a double self-corresponding point. Hence every such quintic scroll must be unicursal. Let the equations of the two conics be

$$c_1: \qquad z = 0, \quad x^2 - yw = 0;$$
$$c_2: \qquad x = 0, \quad z^2 - yw = 0,$$

respectively. The points of each may be expressed parametrically thus:

$$c_1: \qquad x = \lambda, \quad y = \lambda^2, \quad z = 0, \quad w = 1;$$
$$c_2: \qquad x = 0, \quad y = \mu^2, \quad z = \mu, \quad w = 1.$$

The equations of a generator which joins the variable point λ to the variable point μ may be written

$$(1) \qquad \mu x + \lambda z - \lambda\mu w = 0, \quad \lambda x - y + \mu z = 0.$$

The line (1) will generate a unicursal quintic scroll having c_1, c_2 for double conics when λ, μ are connected by a relation of the form described. Since the complete double curve of a unicursal quintic is of order 6 it follows that the residual curve is of order 2. Double generators are impossible since the double element in the (2, 2) correspondence is self-corresponding.

* Ueber die geradlinigen Flächen fünften Grades, *Crelle's Journal*, vol. 67.