SOME INSTRUCTIVE EXAMPLES IN THE CALCULUS OF VARIATIONS.

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IN the following note I propose to give some examples which illustrate in a simple manner several points of fundamental importance in the calculus of variations.

§ 1. *The General Problem and its Assumptions.*

We consider the problem * to minimize the integral

$$I = \int_{x_0}^{x_1} F(x, y, y')dx \qquad (1)$$

under the following *assumptions* :

1. The function $F(x, y, z)$ considered as a function of the three independent variables x, y, z is real and regular † in the vicinity of every finite real point $x = a$, $y = b$, $z = c$ for which $x = a$, $y = b$ lies in a given region R of the xy-plane.

2. The functions $y = f(x)$ admitted to consideration satisfy the following conditions :

(a) For the given end values $x = x_0$, $x = x_1$, y takes the given values $y = y_0$ and $y = y_1$ respectively, *i. e.*, the "curves" $y = f(x)$ pass through two given points $A : (x_0, y_0)$ and $B : (x_1, y_1)$;

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* Compare for the formulation of the problem Osgood's article "Sufficient conditions in the calculus of variations," *Annals of Math.*, 2nd ser., vol. 2 (1901), p. 105. We deviate, however, in several points from Osgood's assumptions.

† *i. e.* developable into an ordinary power series in $x - a$, $y - b$, $z - c$.