NON-EUCLIDEAN GEOMETRY.

Non-Euclidean Geometry. BY HENRY PARKER MANNING, Ph.D.   Boston, Ginn & Co., 1901.   16mo., pp. 95.

THIS work is, as far as we know, the first original book upon this subject that has been published in English.   Perhaps, the adjective " original " is misleading, for the author, in his preface, disclaims any attempt at originality.   His object is to collect a number of the simpler theorems of non-euclidean geometry, and present them in compact and logical form to readers of slight mathematical knowledge.

At the outset, he avoids a dangerous pit-fall : metaphysics.   To pass over in silence all of those philosophical questions which lie at the base of geometry, would be unpardonable in a book of more ambitious nature, but in this case, the omission seems wise.   A beginner can extract but little profit from discussions of the foundations of geometry.   He will accept with equal gladness Russell's contention that the conception of geometrical equality depends upon that of rigid motion, or Veronese's view that rigid motion presupposes a continuous succession of geometrically equal figures. At the same time our author makes a mistake in throwing upon the elementary text books the responsibility for most of the fundamental definitions and assumptions.   We are left in doubt, for instance, whether he considers a line as the path of a moving particle or the boundary of a surface.   It seems also a mistake to lay down, as universally valid, the axiom that two straight lines can meet but once, for spherical geometry is thus excluded.   It is safe to say that spherical geometry is of quite as much importance as elliptic, and the study of such a figure as the sphere with two centers, might well prove attractive to beginners.*   A wiser plan would be to put this axiom among the assumptions for restricted figures, and then prove that if space may be moved as a whole, two straight lines can not intersect more than twice.†

The general arrangement of the book seems to us excellent.   The author first brings out the points of similarity of his three kinds of geometry, then develops their individual peculiarities.   Moreover, the analytic work is placed at the end, out of consideration for those readers who prefer to avoid trigonometry, and are stampeded by the calculus.

---

* Conf. Veronese, Grundzüge der Geometrie von mehreren Dimensionen (German translation by Schepp), p. 503.

† Killing, Grundlagen der Geometrie, p. 56.