MUTH'S ELEMENTARTHEILER.

Theorie und Anwendung der Elementartheiler. Von Dr. P. MUTH. Leipzig, B. G. Teubner, 1899. xvi and 236 pages.

THE work under review covers an important subdivision of general invariant theory, a branch which deserves to be more widely known than it is. No doubt the cause of this neglect has been the lack of a text book, and Dr. Muth's monograph will help to remedy the defect.

The original problem which led to the series of investigations in this theory was that of the canonical reduction of two quadratic forms. In 1829 Cauchy published a paper on the secular inequalities of the planets,* in the course of which he showed that two quadratic forms can in general be reduced to sums of squares of the same variables. He also proved that the latent roots of the family† are all real, in the special case when one of the quadratic forms is positive for all real (non-zero) values of the variable, *i. e.*, is *definite.* Cauchy's results, though not perfectly general, cover most of the cases which occur in the first stages of geometry and dynamics. Jacobi (1834) found similar results by a somewhat different process.‡ Both Jacobi and Cauchy exclude the possibility of equal latent roots appearing in the problem.

The first systematic account of all possible types of two quadratic forms (allowing for equal latent roots) is to be found in Sylvester's paper (1851) on the contact of lines and surfaces of the second order.§ Here we meet with the idea of classification by means of *invariant factors.*‖ Sylvester obtains 4 types of contact for conics, and 12 for quadrics; of course the algebraical possibility that the two forms differ only by a constant factor is trivial in a geometrical

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* Exercices de mathématiques, vol. 4, p. 140 = Oeuvres (2d series), vol. 9, p. 174. Cauchy really discusses only the case when one of the original forms is already a sum of squares.

† If A, B are two given quadratic forms, the system of forms $uA + vB$ (u, v arbitrary parameters) will be called a *family* (as an equivalent for Kronecker's term *Schaar*). The *latent roots* are the values of the ratio $(-u : v)$ for which the determinant of the family vanishes; this determinant will be denoted by $|uA + vB|$.

‡ *Crelle*, vol. 12, p. 1 = Ges. Werke, vol. 3, p. 191.

§ *Phil. Magazine* (4th series), vol. 1, p. 119.

‖ Weierstrass's *Elementartheiler*, and Sauvage's *élémentaire diviseur.* A definition is given in a later footnote on Weierstrass's work.