THEOREMS CONCERNING POSITIVE DEFINITIONS OF FINITE ASSEMBLAGE AND INFINITE ASSEMBLAGE

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THE well known wide disagreement* among workers in assemblage theory as to the meaning that should be attached to certain indispensable terms, such as assemblage, defined assemblage, given assemblage, law of formation, and so on, indicates alike the need and the possibility of a more critical grounding of this doctrine. Here as elsewhere the guiding principle of criticism should be the principle of Parsimony, the so-called Razor of Occam : Entia non sunt multiplicanda præter necessitatem. For the problem is one of simplification, of logical economy, of minimizing hypothesis. The ideal is, in a word, to dispense with the undefined. Now nothing seems clearer than that no amount of criticism, however acute, can completely eliminate the undefined from the foundations of knowledge. For every explicit involves some implicit, and so there must be not merely assumption, but implicit and therefore undefined assumption. Whence appears that a residuum of indetermination and doubt must elude even the most refined and searching analysis, and ultimate simplicity, perfect certainty, absolute rigor, remain beautiful dreams, destined never to be quite realized. Nevertheless the problem admits of closer and closer approximate solution, which, theoretically at least, takes the form of an unending series, namely, of substitutions of simpler (there is no simplest) undefined for the less simple. Unfortunately the process appears to conduct sooner or later from seeming light and certainty into the "frightful shadow-land of metaphysics." But it is only appearance and only seeming, for it is the common lot, consciously or unconsciously, to dwell in that land always. Shadows critical reflection undoubtedly produces, but shadows are incident to the illumination of darkness. Cousin is right : La critique est la vie de la science.

* Cf. J. Tannery : "De l'infini mathématique," *Revue générale des sciences*, vol. 8 (1897). Dedekind : Was Sind und Was Sollen die Zahlen, § 3, Art. 32 ; § 5, Art. 64. G. Cantor : "Sur les ensembles infinis et linéaires de points," *Acta Math.*, vol. 2. Couturat : De l'infini mathématique, Appendice, Note IV. E. Borel : Leçons sur la théorie des fonctions, Chap. I. J. Tannery : Introduction à la théorie des fonctions d'une variable, § 15.