

Similar examples for a larger number of functions can readily be built.

The following theorem however is true :

*If  $u_1, u_2, \dots, u_n$  are single valued functions of the real variable  $x$  defined at every point of a certain interval and having at every point of this interval derivatives of the first  $n - 1$  orders, and if it is possible to strike out the last row and one of the columns of the determinant  $D$  in such a way that there is no point of the interval in question at which the remaining determinant and its derivative both vanish, then if  $D$  vanishes at every point of the interval, the functions  $u_1, u_2, \dots, u_n$  will be linearly dependent throughout this interval.*

This theorem can be readily proved by a slight extension of the method given for instance by Heffter in his book on linear differential equations p. 233.

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[Note added November 2, 1900 : I have just found in Pascal's book on determinants a reference to three papers by Peano (*Mathesis*, vol. 9 (1889), p. 75 and p. 110 ; *Rend. d. Accad. d. Lincei*, ser. 5, vol. 6 (1897), 1° sem., p. 413), in which the question which I have here considered is taken up. My result is however different from Peano's, which states that the identical vanishing of  $D$  is a sufficient condition for linear dependence, provided there is no point at which the first minors corresponding to the elements of the last column all vanish.]

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## REPORT ON THE GROUPS OF AN INFINITE ORDER.

BY DR. G. A. MILLER.

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VARIOUS terms have been employed to designate the smallest elements of which any abstract group is composed. Cayley has called them symbols,\* or symbols of operation. Dyck and many others have called them operations† or operators. Frobenius and others have called them elements.‡ In what follows we shall employ the last one of

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\* Cayley, *Phil. Magazine*, vol. 7 (1854), p. 41.

† Dyck, *Math. Annalen*, vol. 20 (1882), p. 1.

‡ Frobenius, *Crelle*, vol. 86 (1879), p. 218.