etc., these being of weight 4 and 6 respectively.

If the system of equations (2) is equivalent to a single linear differential equation of the second order, all of the invariants vanish, and conversely.

Any system of form (2) can be transformed into another for which $p_{ik} = 0$. This is called the semicanonical form of the system. The subgroup G' of G which leaves this form unchanged is examined. But we can fulfill the further condition $q_{11} + q_{22} = 0$, and a system for which both $p_{ik} = 0$ and $q_{11} + q_{22} = 0$, is said to be in the canonical form. The subgroup G'' of G' which leaves this unchanged is a finite group of very simple form, and has some additional invariants of the form called quadri-derivatives by Forsyth.

Only a few simple results about covariants are mentioned. This and further generalizations are left for a future paper.

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ON LINEAR DEPENDENCE OF FUNCTIONS OF ONE VARIABLE.

BY PROFESSOR MAXIME BÔCHER.

It is ordinarily stated that the identical vanishing of the determinant

$$D = \begin{vmatrix} u_1 & u_2 & \cdots u_n \\ u_1' & u_2' & \cdots u_n' \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \\ u_1^{[n-1]} & u_2^{[n-1]} & \cdots u_n^{[n-1]} \end{vmatrix}$$

is a sufficient* condition for the linear dependence of the functions $u_1(x), u_2(x), \cdots, u_n(x)$. This is perfectly true if the u's are analytic functions of the complex variable x. *This condition is however no longer sufficient if we are dealing with functions of a real variable,* even though these functions possess derivatives of all orders for every real value of x. The truth of this statement will be seen from the example of two functions u_1 and u_2 defined as follows:

$$u_1 = \begin{cases} e^{-1/x^2} & (x \neq 0) \\ 0 & (x = 0) \end{cases} \qquad u_2 = \begin{cases} e^{-1/x^2} & (x > 0) \\ 0 & (x = 0) \\ 2e^{-1/x^2} & (x < 0) \end{cases}$$

* It is of course a necessary condition provided the u's have derivatives of the first $n - 1$ orders.