$w x u$ and $y z u$ then the joining line of the two remaining points of intersection of the sextics so determined will meet $F$ elsewhere in the two lines $w x$ and $y z$.

Cornell University,
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## NO'SE ON THE GROUP OF ISOMORPHISMS.

BY DR. G. A. MILLER.

(Read before the American Mathematical Society, February 24, 1900.)
Let $s_{1}, s_{2}, \cdots, s_{g}$ represent all the operators of a group $G$ and let $t_{a} s_{\alpha}$ correspond to $s_{\alpha}(\alpha=1,2, \cdots, g)$ in any given simple isomorphism of $G$ with itself. It is evident that $t_{a}$ is some operator of $G$. When $G$ is abelian these $t_{a}$ 's must constitute a group $T$ which is isomorphic with G.* In this isomorphism, $\mathrm{t}_{\alpha}$ evidently can not be the inverse of $s_{\alpha}$ unless $s_{\alpha}=1$. As this condition is sufficient as well as necessary, we have

Theorem I.-Every simple isomorphism of an abelian group $A$ with itself may be obtained by $1^{\circ}$ making $A$ isomorphic with one of its subgroups or with itself in such a manner that no operator corresponds to its inverse, and $2^{\circ}$ making each operator of A correspond to itself multiplied by the operator which corresponds to it in the given isomorphism.

The simplest case that can present itself is the one in which the subgroup of $G$, which corresponds to identity of $T$ in the given isomorphism between $G$ and $T$, includes $T$. The resulting simple isomorphism of $G$ with itself must correspond to an operator in the group of isomorphisms of $G$, whose order is equal to the operator of highest order in $T$. When the order of $T$ is an odd prime number $p$, or the double of an odd prime, only one other case can present itself ; viz, the case in which $T$ corresponds to itself, or to its subgroup of an odd prime order, in the given isomorphism between $G$ and $T$. The resulting simple isomorphism of $G$ with itself may clearly correspond to a cyclical group of order $p-1$, or to any one of its subgroups in the group of isomorphisms of $G$. These results lead to the following

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[^0]:    * When $G$ is non-abelian, these $t_{a}$ 's need not constitute a group, as can be seen from the simple isomorphisms of the symmetric group of order 6 with itself.

