If, however, the adjoined of any group G is discontinuous, G itself, and, of course, every group of the same structure is discontinuous. The bilinear form $\mathfrak{F}_a$ is closely related to the adjoined group. In fact, if ϕ_a denotes the matrix of $\mathfrak{F}_a$, the infinitesimal equations of the adjoined are

$$(\alpha_1', \alpha_2', \cdots, \alpha_r) = (1 + \partial t\phi_\alpha)(\alpha_1, \alpha_2, \cdots, \alpha_r);$$

and we have
$$e^{\mathfrak{F}_\beta} e^{\mathfrak{F}_\alpha} = e^{\mathfrak{F}_\gamma}$$

where
$$\gamma_j = \varphi_j(\alpha_1, \cdots, \alpha_r, \beta_1, \cdots, \beta_r) \qquad (j = 1, 2, \cdots, r).$$

CLARK UNIVERSITY,
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PROOF OF THE EXISTENCE OF THE GALOIS FIELD OF ORDER p^r FOR EVERY INTEGER r AND PRIME NUMBER p.

BY PROFESSOR L. E. DICKSON.

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EXISTENCE proofs have been given by Serret* and by Jordan.† The developments used by Serret are lengthy but quite in the spirit of Kronecker's ideas. The short proof by Jordan, however, assumes with Galois the existence of imaginary roots of an irreducible congruence modulo p.

The proof sketched in this note proceeds by induction. Assuming the existence of the $GF[p^n]$, we derive that of the $GF[p^{nq}]$, q being an arbitrary prime number. Since the $GF[p]$ exists, being the field of integers taken modulo p, it will follow that the $GF[p^q]$ exists, and by a simple induction that the $GF[p^r]$ exists for r arbitrary.

We employ the lemma: *A factor of $x^{p^{nm}} - x$, belonging to and irreducible in the $GF[p^n]$, can be of degree m' if and only if m' divides m.* In particular, the irreducible factors of $x^{p^{nq}} - x$ are of degree q or 1. But the product of the distinct‡ linear factors $x - \nu_i$ belonging to the $GF[p^n]$ is $x^{p^n} - x$.

*Algèbre supérieure, 2, pp. 122–142.

† Traité des substitutions, pp. 16, 17.

‡ Two functions belonging to the $GF[p^n]$ are called distinct if one is not the product of the other by a constant, a mark of the field.