ploration may proceed are numerous and attractive. We have only to follow the example set by Laplace, Poisson, Green, Gauss, Maxwell, Kirchhoff, Saint-Venant, Helmholtz, and their eminent contemporaries and successors. In commending the works of these great masters, to the younger members especially of the AMERICAN MATHEMATICAL SOCIETY, I would not be understood as urging the cultivation of pure mathematics less, but rather as suggesting the pursuit of applied mathematics more. The same sort of fidelity to research and the same sort of genius for infinite industry which enabled those masters to accomplish the grand results of the nineteenth century may be confidently expected to achieve equally grand results in the twentieth century.

THE STATUS OF IMAGINARIES IN PURE GEOMETRY.

(Read before the American Mathematical Society, October 28, 1899.)

IN teaching the elements of analytical geometry we are practically forced to allow, even to encourage, a slipshod identification of the field of geometry with the field of algebra. We must all have realized the disadvantages attendant on this course. If ever we have the chance of repairing the error—if error indeed it be at that stage—it is in teaching synthetic geometry; but we can repair it then only if we can establish the existence of imaginary elements without the slightest dependence on algebra. Many books refer to the analogy of algebra as affording sufficient basis, others openly rely on algebraic principles; Chasles, for instance, in the Géométrie Supérieure (pp. 54–57) relies essentially on quadratic equations, whose imaginary roots assure him of the existence of imaginary points.

The two chief books that deal with absolutely pure geometry are those of von Staudt and Reye. It is one of the axioms of modern mathematics that von Staudt placed the doctrine of imaginaries on a firm geometrical basis; but logical and convincing as his treatment is, when patiently studied in all its detail, it yet seems to me hardly practicable as a class-room method.

Von Staudt's primary domain is the visible universe; the elements of his geometry, together with the idea of direction, are an intellectual abstraction from the results of observation. He then extends his domain beyond the visible universe by formal definition; to replace the idea of direction he introduces a set of "ideal points," and