

commutators of this holomorph. When this order is even, the commutator subgroup of the holomorph includes half of the operators of this cyclical group and all of these operators are commutators of this holomorph.

Since $s^{-1}t^{-1}$ is similar to ts and this is similar to st , we observe that the commutator of two operators is similar to the commutator formed by means of one of these operators and the inverse of the other. The preceding results are, in part, supplementary to those contained in the paper "On the commutator groups," BULLETIN, Vol. IV., pp. 135-139.

CORNELL UNIVERSITY.

THE CALCULUS OF GENERALIZATION.

Calcul de Généralisation. Par G. OLTRAMARE, Doyen de la Faculté des Sciences de l'Université de Genève. Paris, A. Hermann, 1899. 8vo, viii + 191 pp.

THIS work is the magnum opus of the venerable dean of the faculty of sciences, of Geneva, who is probably the oldest living pupil of Cauchy. The volume recapitulates and completes the works of the author published during the last twenty years.

Oltramare regards every function as developable in a series of exponentials; thus, a designating an independent variable, he puts

$$\varphi(a) = A_a e^{a\alpha} + A_\beta e^{\beta a} + A_\gamma e^{\gamma a} + \dots,$$

where $\alpha, \beta, \gamma, \dots$ are any constants real or imaginary, in number finite or infinite. He adopts the shorter notation Ge^{au} for the series $\sum A_u e^{au}$, u taking successively the values $\alpha, \beta, \gamma, \dots$, and the equation

$$\varphi(a) = Ge^{au}$$

then expresses that the function $\varphi(a)$ is generated from e^{au} by generalization.

This is in fact an extension of Liouville's generalized derivatives; the latter defined the derivative of index μ of the function $\varphi(a)$ as given by the equation

$$\frac{d^\mu \varphi}{da^\mu} = A_a e^{a\alpha} \alpha^\mu + A_\beta e^{\beta a} \beta^\mu + A_\gamma e^{\gamma a} \gamma^\mu + \dots;$$

and Oltramare proposed to construct a more general calculus* by considering expressions of the form

* See Laisant's introduction to Oltramare's lithographed essay on the calculus of generalization published previously.