THE ASYMPTOTIC LINES OF THE KUMMER SURFACE.

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THE asymptotic lines of the Kummer surface were first determined by Lie* by means of a transformation which establishes a correspondence between two spaces such that to the points of one space correspond the lines of a complex in the other. These curves have been the object of several other investigations,† all of which have treated the problem from the point of view of line geometry.

A very simple solution of the problem can be obtained without the aid of the line geometry by making use of the parametric representation of the Kummer surface in terms of hyperelliptic functions. Among the indefinite number of such representations that are possible, the one most convenient for the purpose is that given by Weber, by which the homogeneous coördinates of a point on the surface are taken proportional to four linearly independent theta functions of two variables u_1, u_2, of the second order and characteristic zero. The equation of a tangent plane can then be written‡

$$\vartheta(u+v)\,\vartheta(u-v)=0 \tag{1}$$

where $\vartheta(w)(=\vartheta(w_1,w_2))$ is a function of the first order and any characteristic, and v_1,v_2 are arbitrary constants. These latter may in fact be regarded as the tangential parameters for the Kummer surface in a sense the exact dual of that in which u_1,u_2 are the point parameters.

The two factors in (1) separately equated to zero represent the same curve, although discriminating between the two branches which pass through the point of contact of the

* S. Lie, "Sur une transformation géométrique," *Comptes rendus*, vol. 71 (1870), p. 579.

† Klein und Lie, "Ueber die Haupttangentencurven der Kummer'schen Fläche," *Monatsber. der Berl. Akad.*, 1870, p. 891 ; Reye, "Ueber die Singularitätenflächen, etc.," *Crelle*, vol. 97, p. 284 ; Segre, "Sur les courbes de tangents principales, etc.," *Crelle*, vol. 98, p. 301 ; Rohn, "Transformation der hyperelliptischen Functionen, etc.," *Math. Annalen*, vol. 15, p. 315.

‡ Cf. Humbert, "Théorie générale, etc.," *Liouville's Journal*, 1893, p. 112.