$$F(z) = \sum_{n=0}^{\infty} a^n z^{n^2}, \quad (|a| < 1),$$

is single-valued, provided $|a|$ is not too large.

The proof is as follows. Evidently

$$\left|\frac{f(z) - f(z')}{z - z'}\right| = \left|1 + \sum_{n=1}^{\infty} \frac{z^{a^n+1} + z^{a^n}z' + \cdots + z'^{a^n+1}}{(a^n+1)(a^n+2)}\right|$$

$$\geqq 1 - \sum_{n=1}^{\infty} \frac{|z|^{a^n+1} + |z|^{a^n}|z'| + \cdots + |z'|^{a^n+1}}{(a^n+1)(a^n+2)}$$

$$\geqq 1 - \sum_{n=1}^{\infty} \frac{1}{a^n+1} = 1 - \frac{1}{a+1} - \left(\frac{1}{a^2+1} + \frac{1}{a^3+1} + \cdots\right)$$

$$> 1 - \frac{1}{a+1} - \frac{1}{a(a-1)} > 0.$$

Hence $$|f(z) - f(z')| > 0,$$ q. e. d.

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NOTE ON THE PERIODIC DEVELOPMENTS OF THE EQUATION OF THE CENTER AND OF THE LOGARITHM OF THE RADIUS VECTOR.

BY PROFESSOR ALEXANDER S. CHESSIN.

IF we put with Professor S. Newcomb*

(1) $$E = ev_1 + e^2v_2 + e^3v_3 + \cdots$$

(2) $$\rho - \log a = e\rho_1 + e^2\rho_2 + e^3\rho_3 + \cdots$$

where E stands for the equation of the center and $\rho = \log r$, then v_i and ρ_i will be of the form

(3) $$iv_i = \tfrac{1}{2}\Sigma k_j^{(i)} \sin j\zeta,$$

(4) $$i\rho_i = \tfrac{1}{2}\Sigma h_j^{(i)} \cos j\zeta,$$

$$(j = i, \quad i-2, \quad i-4, \quad \cdots \quad, -i),$$

* "Development of the perturbative function," Astronomical Papers prepared for the use of the American Ephemeris and Nautical Almanac, vol. 5, part I., p. 12.