ON THE INTERSECTIONS OF PLANE CURVES.

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IN a review of the theory of the "Intersections of plane curves" in the March number of the BULLETIN (pp. 260–273), Professor Charlotte A. Scott has included a full and appreciative criticism of my paper on "Point-Groups in relation to curves" (*Proceedings of the London Mathematical Society* vol. 26 (1895), pp. 495–544). I am exceptionally fortunate in having my work in this subject so clearly described and explained; and I hope I may be allowed to discuss further some interesting points raised in Miss Scott's paper.

For the sake of clearness it may be well to repeat what is meant by excess and defect. If a C_6 and C_7 are drawn through 4 points on a straight line they intersect again in 38 points. These 38 points are such that a C_8 through 35 of them necessarily passes through the remaining 3, and a C_9 through 37 of them necessarily passes through the last, so that the 38 points supply only 35 independent conditions for a C_8, and 37 for a C_9; these properties are expressed by saying that the 8-ic excess of the group of 38 points is 3, and the 9-ic excess is 1. So, in general the n-ic excess r_n of a group of N points is the excess of N over the number of independent conditions that the point group N supplies for n-ics. This number of conditions is therefore $N - r_n$. So also the n-ic defect q_n of the same point group N is the number of independent conditions by which the group falls short in determining an n-ic; in other words, it is the degree of freedom of the general n-ic through the point group N. Hence the formula

$$N - r_n + q_n = \tfrac{1}{2}n(n+3). \tag{1}$$

Of course r_n may be zero; but it is important to bear in mind, if N is a point group derived in some way from the intersection of curves, that r_n is just as likely to be greater than zero as to be zero.

The terms excess and defect are nearly equivalent to the terms suggested by Cayley, viz., postulation for the number $N - r_n$, and postulandum for q_n; but on account of the fact that the numbers $N - r_n$ and q_n are not so convenient for dealing with as r_n and q_n it is preferable to have simple terms for the latter. Excess and defect are complementary