substitutions are commutative. Hence the commutator of two such operators is of order 2.*

6. A Hamilton group of order 2^{α} contains $2^{2\alpha-6}$ quaternion groups as subgroups. All of these have the commutator group of the entire group in common.†

CORNELL UNIVERSITY,
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NOTE ON THE INFINITESIMAL PROJECTIVE TRANSFORMATION.

BY PROFESSOR EDGAR ODELL LOVETT.

(Read before the American Mathematical Society at the Meeting of April 30, 1898.)

It is proposed here to find the form of the most general infinitesimal projective transformation‡ of ordinary space directly from its simplest characteristic geometric property. Geometrically, infinitesimal projective transformations of space are those infinitesimal point transformations which transform a plane into a plane, *i e.*, which leave invariant the family of ∞^3 planes of ordinary space. Analytically, then, the most general infinitesimal projective transformation is the point transformation

$$Uf \equiv \xi(x, y, z) \frac{\partial f}{\partial x} + \eta (x, y, z) \frac{\partial f}{\partial y} + \zeta(x, y, z) \frac{\partial f}{\partial z} \quad (1)$$

which leaves invariant the partial differential equations

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* Cf. Dedekind: loc. cit.

† Cf. Miller: *Comptes Rendus*, vol. 126 (1898), pp. 1406–1408.

‡ In a note on the general projective transformation, *Annals of Mathematics*, vol. 10, No. 1, the forms of the finite projective transformations of ordinary space and those of n-dimensional space are found directly from the conditions for the invariance of the equations $y'' = 0$, $z'' = 0$, which expresses the geometric property that straight line is changed into straight line by these transformations. The form of the general infinitesimal projective transformation of ordinary space is deduced from the finite transformation by the method of Lie. In this derivation three steps are made to intervene, two of which are removed and the other replaced by a simpler one by the method of the present note: 1° two intersecting planes producing the straight line and its property of invariance; 2° the ordinary differential equations of the straight line and the conditions for their invariance; 3° the finite forms of the transformation.