The cylinder and cone may be treated by similar methods.

The preceding theorems suffice to show the validity of Euclid's constructions. It remains to show that the quadratic space is the least space which will suffice for these constructions, and that the restriction of AC and BC to quadratic values is not arbitrary. The rational operations are all required in the addition and subtraction of lines, and the division of lines into any number of equal parts ; the operation of extracting any square root is equivalent to finding a mean proportional and is therefore essential. Moreover any point of the space can be reached by a finite number of operations. The reason that AC and BC should be restricted to quadratic values is : the function of C is to determine a plane through AB, while AB is the fundamental distance ; if C is assumed to be a point of the space we ought to be able to reach C by Euclid's constructions, starting from A and B.

The antithesis of the quadratic space is a space consisting of the points whose rectangular coördinates, in a certain system, are transcendental quantities. It has an infinitely greater number of points than the quadratic space, their multiplicity being of the order of the continuum. Moreover every line or surface of continuous space, except certain cases of a plane, or lines in a plane, parallel to one of the coördinate planes is also a line or surface of this space. We might therefore expect all Euclid's constructions to be possible in it ; such, however, is not the case, as we have already mentioned. Two striking peculiarities of this space are : two circumferences may intersect in a single point ; a circumference may have no center.

YALE UNIVERSITY,
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NIEWENGLOWSKI'S GEOMETRY.

Cours de Géométrie analytique, par B. NIEWENGLOWSKI; avec une *Note sur les transformations en géométrie*, par EMILE BOREL. Paris, Gauthier-Villars, 1894–96. Vol. I., 483 pp ; Vol. II., 292 pp ; Vol. III., 572 pp.

THE work before us, of which the first two volumes deal with plane analytic geometry, the third with analytic geometry of three dimensions, is intended for boys in the *Classes de mathematiques speciales* at the French *Lycées*, at one of the best known of which (the Lycée Louis-le-