LIE'S DIFFERENTIAL EQUATIONS.*

SOPHUS LIE—*Vorlesungen über Differentialgleichungen mit bekannten infinitesimalen Transformationen.* Bearbeitet und herausgegeben von Dr. Georg Scheffers. Leipsic, Teubner. 1891. 8vo, pp. xiv + 568.

These lectures constitute one of the courses of Lie's cycle which has been repeating itself at the University of Leipsic since 1886. The course serves the double purpose of an elementary introduction to the theory of continuous groups and an exposition of how that theory subordinates the various heterogeneous methods of integrating ordinary differential equations to one general method, the key to which is the notion of an infinitesimal transformation, first introduced by Lie at the inception of his theories. The lectures have been edited with the double object of both scientific and pedagogic usefulness. They are so designed that a fourth semester student of a German university is prepared to read them, and they should offer no difficulty to the American reader who is familiar with the processes of the infinitesimal calculus. The numerous problems and illustrative examples drawn from geometry and mechanics commend the book to the private student.

The book falls into five parts: I. The Notions—Infinitesimal Transformation and One Parameter Group of the Plane, chapters 1–4, pp. 1–85; II. Utility of the Notion of Infinitesimal Transformation in Differential Equations of the First Order in Two Variables, chapters 5–9, pp. 86–187; III. One Parameter Groups in Three Variables, chapters 10–13, pp. 187–286; IV. One Parameter Groups and Infinitesimal Transformations in n Variables, Application of these Notions to Differential Equations, chapters 14–20, pp. 286–472; V. Integration of Ordinary Differential Equations of the Second Order which Admit of a One Parameter Group, and Related Problems, chapters 21–25, pp. 473–566.

I. A point transformation is an operation by which a point is carried into the position of a point. Two equations of the form

$$x_1 = \varphi(x, y), \quad y_1 = \psi(x, y), \quad \frac{\partial(\varphi, \psi)}{\partial(x, y)} \neq 0, \qquad (1)$$

φ and ψ being regular analytic functions, are said to determine a point transformation of the plane into itself. If the equations (1) contain a parameter a, they define a family of

* An interesting account of this work, from a somewhat different point of view, was contributed by Professor E. Study to the *Zeitschrift für Mathematik und Physik,* vol. 38 (1893), pp. 185-192.—Editors.