GEOMETRY OF SOME DIFFERENTIAL EXPRESSIONS IN HEXASPHERICAL COÖRDINATES.

BY DR. VIRGIL SNYDER.

(Read before the American Mathematical Society at its Fourth Summer Meeting, Toronto, Canada, August 16, 1897.)

THIS paper is to be regarded as an appendix to my dissertation: Ueber die linearen Complexe der Lie'schen Kugelgeometrie (Göttingen, Kaestner, 1895).

It will simply give an outline of differential geometry, and show its application to the quadratic complex.

Let $x_1, x_2 \cdots x_6$ be any six variables satisfying a homogeneous quadratic identity; these variables may be regarded as the six homogeneous coördinates of the sphere. The form here assumed will be

$$\Pi(x) \equiv \sum_{i=1}^{6} x_i^2 = 0, \tag{1}$$

where two of the coördinates must be imaginary to represent a real sphere.

The geometric meaning of the variables x_i is essentially the same as the $\xi, \eta, \zeta \cdots$ in my classification of Dupin's Cyclides,* from which they can be derived by a linear transformation.

A homogeneous equation of degree n among the variables $x_1 \cdots x_6$,

$$f(x) = 0. \tag{2}$$

in connection with (1) will define a spherical complex of degree n.

In the neighborhood of any given sphere x', belonging to the complex, it can be replaced by a linear complex

$$\sum_{i=1}^{6} \frac{\partial f}{\partial x_i'} y_i = 0, \tag{3}$$

where y_i are running coördinates.

This complex is called a linear tangent complex to $f(x)$ at x'; every sphere has a corresponding tangent complex, and in fact a whole pencil of them. $f(x) = 0$ is not changed by adding any multiple of $\Pi(x)$ to it, hence

* Criteria for nodes in Dupin's cyclides, with a corresponding classification, *Annals of Mathematics*, vol. 11, No. 5, p. 137, June, 1897.