COLLINEATIONS IN A PLANE WITH INVARIANT QUADRIC OR CUBIC CURVES.

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A LINEAR substitution applied to a ternary form will change, in general, the ratios among its coefficients. Unless the substitution is a specialized one, there are three independent linear forms which it leaves unchanged save by a multiplicative constant, but no such forms of higher order except those that are reducible to products of these three linear factors. So much being premised, it is apparent that if any irreducible forms of higher order are unchanged by a particular linear substitution, it must be by reason of some relation among the coefficients of that substitution; and further, that such a relation must be unaltered when this first substitution is transformed by (*not* compounded with) a second. Such relations are expressible in fact by equations involving only the invariants of the first linear substitution. The expression of a conditional relation in invariant form when there is a quadric invariant of the substitution has been effected by integrating a differential equation to determine transcendental invariant forms, then discussing what relations among the parameters will reduce those forms to quadrics. This method is indeed exhaustive; but for the special problem an exhaustive method is not indispensable. It is possible to obtain the invariant conditional equation by quite elementary processes, not only when quadric forms are to be left unchanged, but also when beside the quadrics there are proper cubic invariants. As preliminary to this main theme I will restate well-known formulæ concerning linear invariants and fundamental invariants of a linear substitution or collineation.

A ternary linear substitution or collineation may be represented by the equation (in Clebsch-Aronhold notation):

$$a_x u_a = 0,$$

the equation in line coördinates (u) of the point into which any point (x) is transformed. More explicitly it is written: