NOTES ON THE THEORY OF BILINEAR FORMS.

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ASSOCIATED with any linear substitution

$$x_r = \sum_1^n {}_s\, a_{rs}\, y_s \qquad (r=1, 2, \cdots u)$$

is the bilinear form

$$\sum_1^n {}_r \sum_1^n {}_s\, a_{rs}\, x_s\, u_r,$$

which we may regard as representing the substitution; and in the symbolic system described below we need not distinguish between a linear substitution and the bilinear form associated with it. Thus we may use the same symbol A to denote indifferently the substitution and the bilinear form.*

If B denotes the bilinear form $\sum_1^n {}_r \sum_1^n {}_s\, b_{rs}\, x_s\, u_r$, $A \pm B$ denotes either the bilinear form $\sum_1^n {}_r \sum_1^n {}_s\, (a_{rs} \pm b_{rs})\, x_r\, u_r$, or the linear substitution to which this form corresponds. Further, in this symbolic notation, $A\,B$, which is termed the product of A into B, signifies either the bilinear form $\sum_1^n {}_r \sum_1^n {}_s\, c_{rs}\, x_s\, u_r$, where

$$c_{rs} = \sum_1^n {}_t\, a_{rt}\, b_{ts} \qquad (r, s = 1, 2, \cdots n),$$

or its associated linear substitution, which is obtained by the composition in a definite order of the substitutions A and B. The composition or "multiplication" of bilinear

* In the notation and nomenclature invented by Cayley a linear substitution and a bilinear form are each represented by the square array of its coefficients, the *matrix* of the bilinear form or of the linear substitution; and we do not distinguish in general between a bilinear form and its matrix, or between a linear substitution and its matrix. See "Memoir on the Theory of Matrices," also "Memoir on the Automorphic Linear Transformation of a Bipartite Quadric Function," *Philosophical Transactions*, 1858. Cayley's symbolic notation differs only in unessential features from the system here described, which was invented by Frobenius subsequent to Cayley's investigations.