$[p^n = 2^5]$	$GF[2^5]$	Primitive root γ where $\gamma^5 = 1+\gamma+\gamma^2+\gamma^3$.
	$LCf[2^5]$	32 indices $*$, 0, 1, . . 30.
	$LHCf[2^5-1]$	31 indices 0, 1, . . 30.

[$*$ 0 1 2 3 5 8 10 12 13 14 18 24 25 27 28]$_{16}$
[4 6 7 9 11 15 16 17 19 20 21 22 23 26 29 30]$_{16}$

$LFCf[(2^5-1)/(2-1)]$ 31 indices 0, 1, . . 30.

[0 1 2 3 5 8 10 12 13 14 18 24 25 27 28]$_{15}$

$[p^n = 2^6]$	$GF[2^6]$	Primitive root γ where $\gamma^6 = 1+\gamma+\gamma^3+\gamma^4$.
	$LCf[2^6]$	64 indices $*$, 0, 1, . . 62.
	$LHCf[2^6-1]$	63 indices 0, 1, . . 62.

[$*$ 0 1 2 3 4 6 13 14 16 18 20 21 22 25 26 31 35 37 40 42 43 46 49 50 51 53 54 56 57 58 59]$_{32}$
[5 7 8 9 10 11 12 15 17 19 23 24 27 28 29 30 32 33 34 36 38 39 41 44 45 47 48 52 55 60 61 62]$_{32}$

$LFCf[(2^6-1)/(2-1)]$ 63 indices 0, 1, . . 62.

[First line above, omitting the $*$]$_{31}$

THE UNIVERSITY OF CHICAGO,
August 25, 1895.

ELEMENTARY PROOF OF THE QUATERNION ASSOCIATIVE PRINCIPLE.*

BY PROFESSOR ARTHUR S. HATHAWAY.

THE variety of demonstrations that Hamilton has given of the associative principle of quaternion multiplication, and the remarks that he has made upon such demonstrations, show that he considered an elementary proof of this principle as very desirable. Only two of Hamilton's proofs have been generally employed by subsequent writers—the direct proof by spherical conics, and the indirect one depending upon the assumed laws of i, j, k—and the proof that he considered the most elementary has been entirely ignored, probably because of its deviation from fundamental ideas. On page 297 of the *Elements*, Hamilton calls attention to another method, as follows: "The *associative principle* of multiplication may also be proved without the distributive principle, by certain considerations of *rotations of a system*, on which we cannot enter here."

It is, of course, easy to see that such a proof is possible; but the details of it could not have presented themselves to Hamilton in an elementary form, or he would have seen that it

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