no hesitation, add several well-known names, and many of lesser note, to those mentioned.

The error pointed out in the article in the *Educational Review* can hardly, save on the principle of *ipse dixit*, be called "the most serious error in Cajori's book." It is, however, quite pardonable for Professor Halsted to be partial to that article; he wrote it. As to the article in the *Nation*, that was a charming essay, but it was hardly a serious review of Cajori.

Since with the exception of a few pages in Ball we have no similar attempt at a synopsis of the whole of modern mathematical history up to the present time, it is quite safe to make the sweeping assertion that this portion of the work is "without a rival in the world, in any language." Nevertheless one may close the final chapters with disappointment.

Having no idea of the meaning of Professor Halsted's reference to Chasles' Christian name, or of his statement that "30 is perhaps a slip for 300," the writer ventures to pass them by. He also ventures to reassert his appreciation of the work under discussion as a popular exposition of the historical advance of mathematical science, as set forth in the closing paragraph of his review.

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ON ORTHOGONAL SUBSTITUTIONS.

BY PROF. HENRY TABER.

IN 1846, in *Crelle's Journal*, Cayley gave his well-known determination of the general proper orthogonal substitution of n variables rationally in terms of the minimum number of parameters. Subsequently, in *Crelle*, vol. 50, and in the *Philosophical Transactions* for 1858, Cayley expressed these results in the notation of matrices.

In accordance with the theory of matrices,* two linear substitutions are regarded as susceptible of being added or subtracted. If $(\phi)_{rs}$ denotes that coefficient of the linear substitution ϕ which appears in the rth row and sth column of its square array or matrix, the sum or difference of the linear substitutions ϕ and ψ is defined as follows:

$$(\phi \pm \psi)_{rs} = (\phi)_{rs} \pm (\psi)_{rs}.$$

Addition and subtraction of linear substitutions are then subject to the laws which hold for these processes when we deal with the symbols of ordinary algebra.

* See Cayley's "Memoir on the Theory of Matrices," *Phil. Trans.*, 1858.