NOTE UPON THE HISTORY OF THE RULES OF CONVERGENCE IN THE EIGHTEENTH CENTURY.*

BY PROF. G. ENESTRÖM.

IN the introduction to his interesting article "Evolution of Criteria of Convergence" (BULLETIN OF THE NEW YORK MATHEMATICAL SOCIETY, vol. 2, 1892, pp. 1–10), Professor Cajori mentioned several authors previous to the present century who had recognized the necessity of verifying the convergence of a series before making use of it in mathematical investigations. In this brief note I beg to call attention to two other mathematicians, Maclaurin and Stirling, who had considered rules of convergence; the first of these deserves, in my opinion, a signal place in the history of these rules.

In fact, the fundamental theorem of convergence attributed (l. c., p. 3) by Professor Cajori to Cauchy was established previously by Maclaurin in article 350 of his "Treatise on Fluxions" (Edinburgh, 1742). It is true that Maclaurin enunciated the rule in a geometrical form, but it is very easy to translate it into algebraic language. The fact that this rule is due to Maclaurin was pointed out, I believe, by Montucla, but it does not seem to have been sufficiently appreciated until quite recently. I called attention to it in the *Bibliotheca Mathematica*, 1886, p. 48, and it was mentioned later by R. Reiff in his "Geschichte der unendlichen Reihen" (Tübingen, 1889, p. 120), as well as by W. W. R. Ball in the *second* edition of his "Short Account of the History of Mathematics" (London, 1893, p. 394).

As regards Stirling, he demonstrated in his "Methodus Differentialis" (London, 1730) the following theorem: Let

$$W = w_0 \,.\, w_1 \,.\, w_2 \,.\, w_3 \ldots$$

and

$$w_x = \frac{1}{m}\,\frac{z^\theta + az^{\theta+1} + bz^{\theta+2} + \ldots}{z^\theta + cz^{\theta+1} + dz^{\theta+2} + \ldots},$$

then

$$W = 0, \text{ if } m>1, \text{ or if } m = 1 \text{ and } a<c;$$

$$W = \text{a finite quantity} \gtrless 0, \text{ if } m = 1,\ a = c;$$

$$W = \infty, \text{ if } m<1, \text{ or if } m = 1 \text{ and } a>c.$$

* Translated by the Editors.