

I say, however, that a monogenic function of  $x + iy$  can be formed by aid of *any* two solutions of Laplace's equation and without any quadrature. Suppose  $P$  and  $Q$  to be any two such solutions; they do not then in general satisfy equations (1). If they do satisfy (1), then  $P + iQ$  is the function sought. If they do not satisfy (1), write

$$Q_1 = \frac{\partial P}{\partial x} - \frac{\partial Q}{\partial y},$$

$$P_1 = \frac{\partial P}{\partial y} + \frac{\partial Q}{\partial x}.$$

Then  $P_1 + iQ_1$  is a monogenic function of  $x + iy$ , for

$$\frac{\partial P_1}{\partial x} - \frac{\partial Q_1}{\partial y} = 0,$$

$$\frac{\partial P_1}{\partial y} + \frac{\partial Q_1}{\partial x} = 0,$$

$$\nabla P_1 = \nabla Q_1 = 0,$$

$$\text{since } \nabla P = \nabla Q = 0.$$

## LAMBERT'S NON-EUCLIDEAN GEOMETRY.

BY PROF. GEORGE BRUCE HALSTED.

IN the discussion which followed my lecture in the Mathematical Section of the Congress at Chicago, Professor Study of Marburg mentioned that there had recently been brought to light an old paper of Lambert's on what was long after named the non-Euclidean geometry. Professor Klein jotted down on my Lobatschewsky programme the address of Dr. Staeckel, as the person from whom I might hope for definite information; and from his answer to my letter I extract the following highly interesting facts.

This essay of Lambert's bears the title: "Zur Theorie der Parallellinien." It is dated September, 1766, but was first published in 1786 from the papers left by F. Bernoulli, a relative of John Bernoulli. It appeared in the