NOTE ON THE DEFINITIONS OF LOGARITHM AND EXPONENTIAL.

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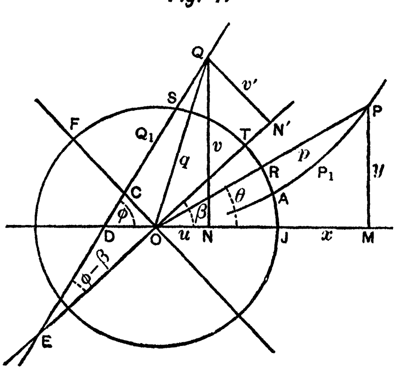
PROFESSOR HASKELL'S interesting and important investigation calls for a modification of my former definitions of the logarithmic and exponential functions.* I propose that the new specifications shall be as follows:

In a circle whose radius is unity OT is fixed and makes an angle β with the real axis, OR turns about O with a constant speed, Q moves along any line ES in the plane with a constant speed, P along OR with a speed proportional to its distance from O.

Following the notation of my former paper, let the velocities of P, Q, R be:

Fig. 4.



$$
\begin{aligned}
&P \text{ in } OR \text{ at } A = \lambda,\\
&Q \text{ in } ES = \mu,\\
&R \text{ in } JRS = \omega;\\
&\text{and let arc } JR = \theta,\\
&\text{circular measure of } ODS = \phi,\\
&OP, OQ = p, q,\\
&ON, NQ = u, v,\\
&ON', N'Q = u', v',\\
&OM, MP = x, y.
\end{aligned}
$$

In all logarithmic systems the relation

$$\frac{\omega}{\lambda} = \tan(\phi - \beta)$$

is assumed to exist. This fixes the ratio of the radial to the transversal velocity of P and determines the *form* of the curve upon which P moves. Its *position* may then be determined by fixing two points upon it, and for this purpose we may assume that P crosses the real axis OJ at the instant Q crosses the line ET, and that P crosses the circumference of the unit circle at the instant Q crosses the line OF which is drawn through the origin perpendicular to ET. As a consequence of these two assumptions, when ES passes through the origin, A coincides with J, and the figure becomes identical with that of my former paper (*loc. cit.* p. 187), which

* *American Journal Mathematics*, vol. 14, p. 187.